MINISTRY OF INDUSTRY AND TRADE

MINISTRY OF EDUCATION AND TRAINING

NATIONAL RESEARCH INSTITUTE OF MECHANICAL ENGINEERING

NARIME

TRAN SI KIEN

DYNAMIC AND OPTIMAL MOTION CONTROL OF INDUSTRIAL MANIPULATORS BASED ON PONTRYAGIN’S MAXIMUM PRINCIPLE

SUMMARY OF DOCTORAL THESIS

Industry: Mechanical Engineering

Item No: 9520103

Hanoi - 2026

The project was completed at:

National Research Institute Of Mechanical Engineering – Ministry of Industry and Trade

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Phản biện 2:

Phản biện 3:

The thesis is defended before the Institute-level doctoral thesis evaluation council

Meeting at: National Research Institute Of Mechanical Engineering – Ministry of Industry and Trade

Head Office Building: No. 4 Pham Van Dong Street, Phu Dien Ward, Hanoi City

At .... (hour), on ……. (month) …….. (day) ………. (year)

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INTRODUCTION

1. Urgency of the topic

In recent years, along with the rapid development of automation, smart manufacturing, and digital transformation in industry, industrial manipulators have been increasingly widely used in various fields, such as assembly, welding, painting, machining, loading and unloading, packaging, and material handling.

In practical operation, the requirements imposed on industrial manipulators are not limited to their ability to perform motions according to technological demands, but also include criteria related to accuracy, stability, flexibility, working cycle time, and energy efficiency.

Dynamic analysis of industrial manipulators provides the basis for determining the relationship between the motion of the system and the forces and moments acting on its links, joints, and drive mechanisms. Although numerous studies have been conducted on kinematic modeling, dynamic modeling, and control of manipulators, this remains a field in which many issues require further clarification.

In addition, in many current studies and applications, numerical simulation plays an increasingly important role in the design, evaluation, and optimization of industrial manipulator motion. At the same time, simulation models and data can also provide a reference basis for further research directions related to intelligent control, data-driven optimization, and digital twin models in the field of industrial robotics.

From the above analysis, it can be seen that research on dynamic analysis and optimal motion control of industrial manipulators is necessary.

2. Research objectives

Nghiên cứu cơ sở lý thuyết và xây dựng khung phương pháp phân tích động lực học, tối ưu hóa chuyển động của tay máy công nghiệp, làm cơ sở cho việc mô phỏng và đánh giá một số bài toán chuyển động điển hình.

3. Research object and scope

3.1. The research object

The research object of the dissertation is open-chain industrial manipulators. The specific objects investigated in the numerical simulations are models of planar manipulators with three degrees of freedom.

3.2. Research scope

The dissertation focuses on studying industrial manipulators in terms of mechanical modeling, dynamic analysis, and motion optimization.

- The links of the manipulator are assumed to be perfectly rigid bodies, without considering link and joint elasticity or uncertainty-related issues, such as parameter errors and payload disturbances.

- The types of motion considered for optimization include point-to-point motion and motion along a prescribed trajectory of the end-effector.

- The selected optimization criterion is the reduction of control effort.

4. Research method

The dissertation employs a theoretical research method combined with numerical simulation. Based on multibody system mechanics and optimal control theory, the problems of dynamic analysis and motion optimization of industrial manipulators are formulated and investigated through modeling, equation establishment, and the solution of optimization problems using numerical simulation tools.

5. Scientific and practical significance

5.1. Scientific significance

- The study develops a theoretical foundation and methodological framework for the optimal control problem of industrial manipulators.
- The optimal control-effort trajectories are determined for several planar manipulators, thereby contributing to clarifying the applicability of the approach based on Pontryagin's principle in research on the motion optimization of industrial manipulators.

5.2. Practical significance

- The proposed research method can support the selection of motion schemes aimed at reducing control effort and improving the working efficiency of manipulators in industrial operation processes.
- The results of the dissertation can provide a reference basis regarding input parameters, dynamic models, cost functions, and simulation data for further studies on intelligent control and data-driven robot motion optimization.

6. Novel contributions of the dissertation

- A methodological framework is proposed for the problems of dynamic analysis and optimal motion control of industrial manipulators. The methodological components are organized into a unified procedure that directly supports the formulation and solution of optimal motion control problems for industrial manipulators.
- Optimal control problems are formulated for several typical motion types of industrial manipulators, with direct consideration of the dynamic structure of the manipulator rather than treating them merely as separate kinematic trajectory-planning problems.
- The optimal control laws are verified through numerical simulations for several representative planar manipulator models. Quantitative comparisons are made in terms of the optimization criterion against several motion laws and other methods, thereby illustrating the applicability of the proposed method.

CHAPTER 1. OVERVIEW OF THE RESEARCH PROBLEM

1.1. General introduction of industrial manipulators

Industrial manipulators are an important component of modern automation systems. According to ISO terminology, an industrial robot is understood as an automatically controlled, reprogrammable, multipurpose actuator that operates along three or more axes and may be fixed in place or mounted on a mobile platform to serve automation applications in industrial environments [1].

1.1.1. Sự phát triển của tay máy công nghiệp

The development of industrial manipulators is closely associated with the transition from hard automation to flexible automation and smart manufacturing.

In terms of deployment scale, data from the International Federation of Robotics indicate that industrial robots have maintained a long-term growth trend over the past decade. According to the World Robotics report, the number of newly installed industrial robots worldwide has exceeded 500,000 units per year for several consecutive years, while the total number of robots in operation worldwide has reached more than 4 million units [2], [4].

A notable trend in recent years is the parallel development of traditional industrial robots and collaborative robots.

1.1.2. Advantages of industrial manipulators

Industrial manipulators offer numerous advantages in production and manufacturing.

First, they can operate with high repeatability and maintaining stable operational quality over long periods of time.

Second, they can improve the productivity and flexibility of production systems.

Third, they contribute to improving working conditions and production safety.

In addition to the advantages mentioned above, an increasingly important benefit is the ability to optimize operational efficiency, particularly with respect to cycle time, energy consumption, and equipment durability.

1.2. Overview of dynamic analysis and motion optimization of industrial manipulators

Dynamic analysis and motion optimization are two central aspects of modern research on industrial manipulators. They are closely related and complement each other throughout the processes of modeling, control, and evaluation of operational efficiency [6], [7], [9], [10].

While dynamic analysis answers the question of “what force or moment is required for the manipulator to move in a given manner,” motion optimization addresses the question of “how the manipulator should move in order to be optimal according to a specified criterion.” Motion optimization is a broad concept that encompasses trajectory planning, motion planning, time allocation, optimization of energy-related criteria, and optimal control [7], [10], [12].

1.3. Research situation worldwide and in Vietnam

1.3.1. Dynamic analysis of robot manipulators

Worldwide, studies on the dynamic analysis of robot manipulators have mainly developed along classical methodological directions, among which the

Newton–Euler method, the Lagrange method, and the Hamilton method are prominent [13]–[16].

In general, it can be observed that the methods for dynamic analysis of robot manipulators have been established relatively comprehensively, and their effectiveness has been verified through numerous studies. Therefore, in recent years, international studies have focused less on re-evaluating these fundamental methods independently and have instead mainly developed toward extensions and more specialized applications.

1.3.2. Motion optimization of robot manipulators

1.3.2.1. Overview of approaches to robot manipulator motion optimization

Motion optimization of robot manipulators is a research direction that has received attention for a long time and has continued to develop in response to increasing demands for productivity, accuracy, and operational efficiency of industrial robots.

Based on published studies, the main approaches to the motion optimization problem of robot manipulators can be classified into three major groups: motion planning, direct methods, and indirect methods.

Another noteworthy research trend is the shift from purely time-optimal problems to multi-objective optimization problems. However, the diversity of criteria and approaches has also led to a fragmentation of studies according to specific problem types, objective functions, and formulations of the optimization problem.

1.3.2.2. Studies based on the motion planning approach

In general, studies based on the motion planning approach are mainly valuable in establishing a foundation for the motion optimization problem of robot manipulators based on motion optimization theory. However, when considering multi-degree-of-freedom industrial manipulators with nonlinear dynamic models and motion problems involving boundary conditions or specific trajectory constraints, motion planning often faces major obstacles in terms of computational feasibility. This is an important reason why, despite its rigorous theoretical foundation, motion planning has not become the dominant approach in most current studies on the optimal motion of industrial manipulators.

1.3.2.3. Studies based on direct methods

In essence, with this approach, the original optimal control problem is not handled by directly deriving the necessary optimality conditions but is instead transformed into a finite-dimensional optimization problem by parameterizing the state trajectory or the control signals. The resulting problem is then solved using numerical optimization tools, often in the form of nonlinear programming [18], [32]. Some common limitations are that the solution quality depends strongly on the choice of algorithmic parameters, the computational time may be substantial, and it is difficult to provide a rigorous theoretical assessment of the optimality of the solution.

1.3.2.4. Studies based on indirect methods

From the published works, it can be observed that indirect methods have demonstrated the ability to handle many classes of optimal control problems. A notable advantage of this approach is that it clarifies the structure of the optimization problem based on the dynamic model, thereby directly establishing the relationship among the state variables, costate variables, control variables, and boundary conditions. For problems in which the optimality criterion is closely associated with joint forces or moments, this represents an important advantage.

1.3.2.5. Studies in Vietnam

One application-oriented work is the study by Le Anh Kiet and co-authors in 2016 [63] on the design and fabrication of a control system for an AKB loading and unloading robot.

In the direction of optimizing control parameters to improve response quality, Duong Xuan Bien and Chu Anh My in 2018 [64] studied the design problem of a controller for a two-link rigid manipulator. In this work, the parameters of the PID controller were optimized using particle swarm optimization (PSO) to improve positioning accuracy.

From the perspective of kinematic motion optimization, Nguyen Mai Quyen, Chu Binh Minh, and Ha Binh Minh in 2022 [65] presented a method for calculating and optimizing joint angles in the control of a planar three-degree-of-freedom robotic arm. The problem was formulated as determining the joint angles such that the end-effector moves along the prescribed trajectory.

Luu Thi Hue and Nguyen Pham Thuc Anh in 2022 [57] proposed a method for designing the optimal trajectory for robots using an adaptive genetic algorithm.

1.3.2.6. Remarks on the research situation of robot manipulator motion optimization

Based on the reviewed studies worldwide, the problem of robot manipulator motion optimization has been investigated using various approaches and has achieved significant results.

Compared with international studies, domestic research on motion optimization and optimal control of robot manipulators remains more limited, both in terms of the number of publications and the diversity of approaches.

1.4. Research orientation of the dissertation

From the above remarks, several research gaps on which the dissertation will focus can be identified as follows:

- First, an approach is needed that closely links the dynamic model of industrial manipulators with the motion optimization problem, rather than being limited to trajectory optimization in a purely kinematic sense or to purely numerical optimization.

- Second, typical motion cases of industrial manipulators need to be investigated in order to evaluate the applicability of the proposed methodological framework in practically meaningful situations.

The research contents of the dissertation are as follows:

- Studying the theoretical foundations of manipulator dynamics and optimal control.
- Developing dynamic models and formulating optimal control problems for industrial manipulators using Pontryagin's principle.
- Numerically simulating the optimal control of manipulators for typical motions.

Conclusion of Chapter 1

From the contents of Chapter 1, several conclusions can be drawn as follows:

1- From the review of domestic and international research, the motion optimization of industrial robots continues to attract the attention of researchers.

2- The theory of dynamic analysis of industrial robots has been well established, with major methods such as Newton–Euler, Lagrange, Hamilton, and approaches for constrained manipulators. These methods provide the foundation for modeling and investigating manipulator motion. For the problems of motion optimization and optimal control of robot manipulators, several main approaches have been developed, including motion planning, direct methods, and indirect methods.

3- The dissertation defines its research content as using dynamic models based on the Lagrange method and appropriate principles to formulate and solve the motion optimization problem of industrial manipulators through an indirect method based on Pontryagin's principle. This approach is applied to several typical motion cases to clarify the optimal control laws of the system.

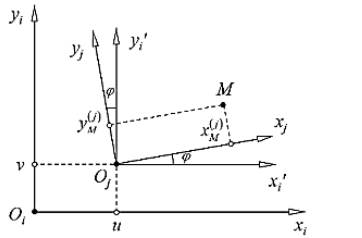
CHAPTER 2. THEORETICAL FOUNDATIONS OF DYNAMIC ANALYSIS AND OPTIMAL CONTROL OF INDUSTRIAL MANIPULATORS

2.1. Theoretical foundations of manipulator dynamics

2.1.1. Homogeneous transformation matrix method

Consider a point M whose position in the coordinate system $O_j x_j y_j$ is given by $\mathbf{r}_M^{(j)} = \begin{bmatrix} x_M^{(j)} & y_M^{(j)} \end{bmatrix}^{\mathrm{T}}$, its position in the coordinate system $O_i x_i y_i$ is

$$\begin{aligned} x_M^{(i)} &= u + x_M^{(j)} \cos\varphi - y_M^{(j)} \sin\varphi \\ y_M^{(i)} &= v + x_M^{(j)} \sin\varphi + y_M^{(j)} \cos\varphi \end{aligned} \qquad (1.1)$$



Hình 2.1. Biểu diễn các hệ tọa độ và vị trí điểm M

Define the transformation matrices

$$\mathbf{t}_u = \begin{bmatrix} 1 & 0 & u \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \mathbf{t}_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & v \\ 0 & 0 & 1 \end{bmatrix}; \mathbf{t}_\varphi = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1.2)$$

The matrix $\mathbf{T}_j^i = \mathbf{t}_u \mathbf{t}_v \mathbf{t}_\varphi$ is also called the transformation matrix from the coordinate system $O_j x_j y_j$ to the coordinate system $O_i x_i y_i$.

2.1.2. Application of the homogeneous transformation matrix method to manipulators

At that time, the matrix representing the transformation from the coordinate system $O_i x_i y_i$ to the fixed coordinate system $O_0 x_0 y_0$ is:

$$\mathbf{T}_i^0 = \mathbf{T}_1^0 \mathbf{T}_2^1 \mathbf{T}_3^2 \ldots \mathbf{T}_i^{i-1} \tag{1.3}$$

The coordinates of the end-effector point E in the fixed coordinate system are then expressed as:

$$\begin{bmatrix} \mathbf{r}_E^{(0)} \\ 1 \end{bmatrix} = \mathbf{T}_n^0 \begin{bmatrix} \mathbf{r}_E^{(n)} \\ 1 \end{bmatrix} \tag{1.4}$$

For each manipulator model, the transformation matrices $\mathbf{T}_1^0, \mathbf{T}_2^1, \ldots$ are determined, and the matrix $\mathbf{T}_n^0$ is then obtained. Subsequently, the position, velocity, and acceleration of the end-effector point are determined.

2.2. Theoretical foundations of manipulator dynamics

2.2.1. Matrix form of the second-kind Lagrange equations

Phương trình Lagrange loại 2 của tay máy với $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \ldots & q_n \end{bmatrix}^T$ denoting a complete set of generalized coordinates, are written as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = -\frac{\partial P}{\partial q_i} + Q_i^{kt} \;\; ; \;\; i = 1, 2, \ldots, n \tag{1.5}$$

where T is the kinetic energy of the system, P is the potential energy function, and Q_i^{kt} is the generalized force corresponding to non-conservative forces.

The dynamic equations of the manipulator can be expressed in matrix form as follows [1]:

$$\mathbf{A}\ddot{\mathbf{q}} = \mathbf{Q} + \mathbf{Q}_{qt} \tag{1.6}$$

2.2.2. Equations of motion of constrained manipulators

In many technological problems, robot manipulators do not move completely freely within the workspace but must additionally satisfy certain kinematic or dynamic constraints.

In matrix form, the equation of constraint reaction forces is given by

$$\mathbf{D}\mathbf{Q}^* = \mathbf{0} \tag{1.7}$$

with $\mathbf{D} = \left[d_{\sigma j} \right]$ is a matrix of size $(n - r) \times n$ and $\mathbf{Q}^* = \begin{bmatrix} Q_1^* & Q_2^* & \ldots & Q_n^* \end{bmatrix}^T$.

At this point, the matrix-form equations of motion are established as

$$\mathbf{D}\mathbf{A}\ddot{\mathbf{q}} = \mathbf{D}\left(\mathbf{Q} + \mathbf{Q}^{qt}\right) \tag{1.8}$$

The system of equations (2.19), together with the constraints, forms a system of differential-algebraic equations, but only n generalized redundant coordinates remain.

2.3. Theoretical foundations of optimal control of manipulators

2.3.1. Optimal control problem of manipulators

Consider a robot manipulator as a dynamic system whose differential equations of motion are expressed in the form

$$\dot{\mathbf{x}}(t)=\mathbf{f}\left(\mathbf{x}(t),\mathbf{u}(t)\right),\ \mathbf{x}(t_0)=\mathbf{x}_0,\ \mathbf{x}=\begin{bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{bmatrix} \tag{1.9}$$

In general form, the objective function can be written as

$$J=\phi\left(\mathbf{x}_f\right)+\int_{t_0}^{t_f}L\left(\mathbf{x}(t),\mathbf{u}(t)\right)\mathrm{d}t \tag{1.10}$$

In general form, the terminal conditions can be written as

$$\mathbf{\Phi}\left(\mathbf{x}_f\right)=0 \tag{1.11}$$

From the above components, the optimal control problem of a robot manipulator can be stated in general form as follows::

Consider a robot manipulator whose differential equations of motion are expressed in the form (2.20). Determine the control variables $\mathbf{u}^*(t)$ *such that the system moves from the initial state* $\mathbf{x}(t_0)=\mathbf{x}_0$*, satisfies the terminal conditions (2.23), and, at the same time, the objective function (2.22) attains its optimal value.*

2.3.2. Pontryagin's Maximum Principle

To construct the necessary conditions for the optimal problem, an auxiliary set of variables, called costate variables, is introduced. The costate vector is denoted by **p**.

The Hamiltonian function, which combines the dynamics of the system and the cost, is constructed as follows:

$$H\left(\mathbf{x},\mathbf{u},\mathbf{p}\right)=\mathbf{p}^{\mathrm{T}}\mathbf{f}\left(\mathbf{x},\mathbf{u}\right)+p_0L\left(\mathbf{x},\mathbf{u}\right);\ p_0\in\{0,-1\} \tag{1.12}$$

Pontryagin's principle:

A necessary condition for the optimal control $\mathbf{u}=\mathbf{u}^*(t)$ to transfer the system (2.20) from the initial state $\mathbf{x}(t_0)=\mathbf{x}_0$, satisfy the terminal conditions (2.23), follow the corresponding optimal trajectory $\mathbf{x}=\mathbf{x}^*(t)$, have the corresponding costate variable $\mathbf{p}=\mathbf{p}^*(t)$ and make the objective functional (2.22) attain the required extremum is as follows:

1. There exists an extended costate vector $\mathbf{p}=\left[p_0,\mathbf{p}\right]^{\mathrm{T}}$ which is not identically zero and satisfies the costate equation and the state equation.

2. The Hamiltonian function $H(\mathbf{x}, \mathbf{u}, \mathbf{p})$ attains its maximum with respect to $\mathbf{u}^*$

3. At the terminal time $t = t_f$, the transversality condition for the costate variables is satisfied.

$$\mathbf{p}(t_f) = \left[\frac{\partial \phi(\mathbf{x}_f)}{\partial \mathbf{x}}\right]^{\mathrm{T}} + \left[\frac{\partial \mathbf{\Phi}(\mathbf{x}_f)}{\partial \mathbf{x}}\right]^{\mathrm{T}} \boldsymbol{\psi} \tag{1.13}$$

2.3.3. Application of Pontryagin's principle to optimal control of manipulators

The dynamic equations can be written in the state space as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{A}^{-1}(\mathbf{q})\left[\mathbf{u} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})\right] \end{bmatrix} \tag{1.14}$$

Thus, the application of Pontryagin's principle to the optimal control problem of robot manipulators leads to a system consisting of the state equations, costate equations, extremum condition, and boundary conditions.

2.4. Theoretical foundations of several motion laws for quantitative comparison

2.4.1. Cubic polynomial motion law

Consider a motion variable over the time interval from t_0 to t_f. This variable is described by a cubic polynomial in time as:

$$\varphi(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{1.15}$$

where $\varphi(t)$ is the motion variable, and a_0, a_1, a_2, a_3 are coefficients to be determined.

2.4.2. Quintic polynomial motion law

Consider a motion variable over the time interval from t_0 to t_f. This variable is described by a quintic polynomial in time as:

$$\varphi(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \tag{1.16}$$

where $\varphi(t)$ is the motion variable, and $a_i, i = \overline{0..5}$ are coefficients to be determined.

2.4.3. S-curve motion law

Consider a motion variable over the time interval from t_0 to t_f. he jerk of the motion variable is defined as:

$$j(t) = \dddot{\varphi}(t) = \frac{\mathrm{d}^3\varphi(t)}{\mathrm{d}t^3} \tag{1.17}$$

In a seven-phase S-curve motion law, the motion is divided into seven consecutive time intervals..

2.4.4. Procedure for comparing motion laws according to the objective function J

For the point-to-point motion problem in joint space, the reference motion laws are constructed directly for each joint variable:

$$q_i(t) = \varphi_i(t), \; i = 1, 2, ..., n \tag{1.18}$$

For the problem with an end-effector trajectory constraint, consider the end-effector trajectory described in parametric form as:

$$x_E = x_E(\varphi), \; y_E = y_E(\varphi) \tag{1.19}$$

Accordingly, $\mathbf{q}(t), \dot{\mathbf{q}}(t), \ddot{\mathbf{q}}(t)$ are substituted into the dynamic equations to determine the corresponding forces or moments.

2.5. Proposed methodological framework

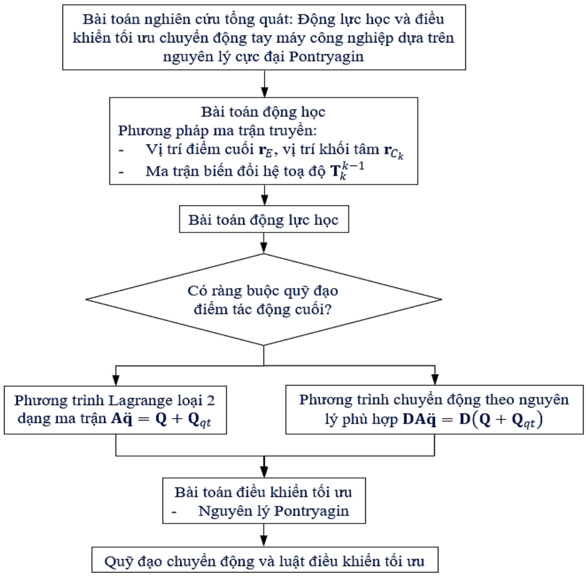


Fig. 2.1. Proposed methodological framework

Conclusion of Chapter 2

From the contents of Chapter 2, several conclusions can be drawn as follows:

1- The problem of dynamic analysis and motion optimization of industrial manipulators can be established on a unified theoretical basis. This connection shows that the three layers of content are not separate from one another, but instead form a complete logical sequence of research.

2- The selection of the homogeneous transformation matrix method, the matrix form of the second-kind Lagrange equations, combined with appropriate principles and Pontryagin's principle, is consistent with the research orientation of the dissertation.

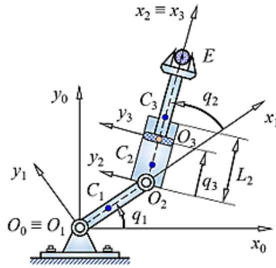
3- A general research framework has been developed for the dissertation with respect to the class of problems involving dynamic analysis and motion optimization of industrial manipulators.

CHAPTER 3. OPTIMAL CONTROL OF INDUSTRIAL MANIPULATORS IN POINT-TO-POINT MOTION

3.1. Optimal control of a manipulator moving to a target point

3.1.1. Problem statement

Consider a planar manipulator with two revolute links and one prismatic link at the end, namely a planar RRP manipulator, as shown in Figure 3.1.



Hình 3.1. Tay máy phẳng RRP dạng 1

The problem is to reduce the control effort, or to optimize the control effort of the manipulator, by determining the optimal driving forces/moments at the joints, namely the control variables **u**, such that the end-effector E moves from the initial point (x_0, y_0), corresponding to the robot manipulator configuration $\mathbf{q}(0) = \mathbf{q}_0$, to the final point ($x_f$, y_f), corresponding to the robot manipulator configuration $\mathbf{q}(t_f) = \mathbf{q}_f$, within the prescribed time tf..

3.1.2. Establishment of the dynamic equations

3.1.3. Formulation of the optimal control problem

The requirement is to determine the control variables $\mathbf{u}(t)$ and the corresponding motion law in joint space $\mathbf{q}(t)$ such that the end-effector of the manipulator moves from the initial position $M_0(x_0, y_0)$, corresponding to the initial configuration $\mathbf{q}_0 = \mathbf{q}(0)$, to the target position $M_f(x_f, y_f)$, corresponding to the final configuration $\mathbf{q}_f = \mathbf{q}(t_f)$. The quantities $\mathbf{q}_0, \mathbf{q}_f, t_f$ are prescribed, and the optimality criterion is to minimize the control effort, represented by the time integral of the squared control forces/moments:

$$J = \int_0^{t_f} f_0 \mathrm{d}t \to \min\ ;\ f_0 = \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2\right) \qquad (1.20)$$

3.1.4. Simulation results for optimal control

a) Simulation setup

Manipulator parameters:

$m_1 = 4,8; m_2 = 2,4;\ m_3 = 3,0;\ m = 1,5\ [\mathrm{kg}]$

$L_1 = 1,0; L_2 = 0,5; L_3 = 0,6\ [\mathrm{m}];\ c_1 = 0,5;\ c_2 = 0,25;\ c_3 = 0,3\ [\mathrm{m}]$

$$J_1 = 0,4;\; J_2 = 0,05;\; J_3 = 0,09\; \left[\text{kg.m}^2\right]$$

$$b_1 = 5;\; b_2 = 5\left[\text{N.m.s/rad}\right];\; b_3 = 5\left[\text{kg/s}\right];\; g = 10\left[\text{m/s}^2\right];\; t_f = 3,0\left[s\right]$$

The initial and final configurations of the manipulator are selected as follows:

$$M_0 : q_1(0) = 0;\; q_2(0) = 0,2\; [rad];\; q_3(0) = 0,1[m]$$

$$M_f : q_1(t_f) = 1,4;\; q_2(t_f) = 1,0\; [rad];\; q_3(t_f) = 0,5[m]$$

The boundary conditions for the joint velocities are selected as follows:

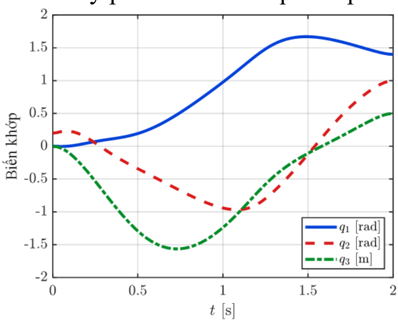
$$\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = \dot{q}_1(t_f) = \dot{q}_2(t_f) = \dot{q}_3(t_f) = 0$$

The objective functional is: $z = \int_0^t f_0 \mathrm{d}t$ với $z(0) = 0$.

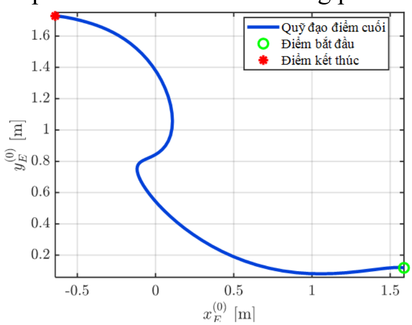
The boundary-value system is solved using MATLAB® software.

b) Simulation results

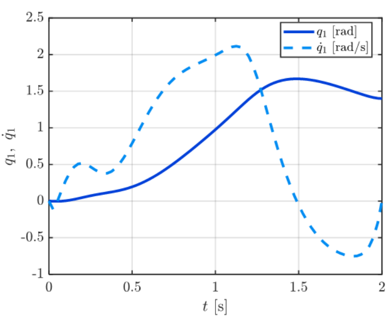
In Figure 3.2, the joint variables vary continuously over time, and the joint angle values at the initial and final times coincide with the initial and target points specified in the boundary conditions. In addition, the motion trajectory of the end-effector shown in Figure 3.3 is a continuous curve, indicating that the manipulator successfully performs the required point-to-point motion in the working plane.



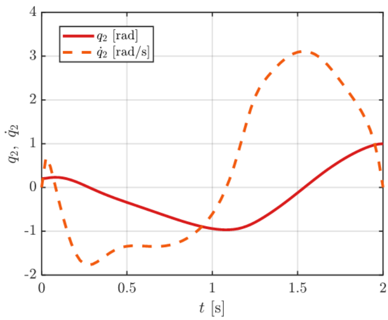
Hình 3.2. Đồ thị các biến khớp



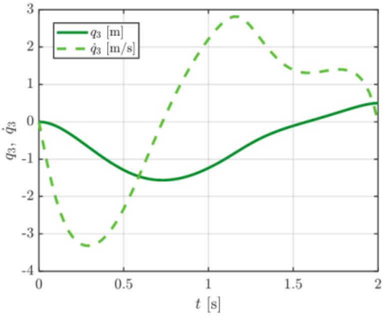
Hình 3.3. Đồ thị quỹ đạo chuyển động điểm cuối



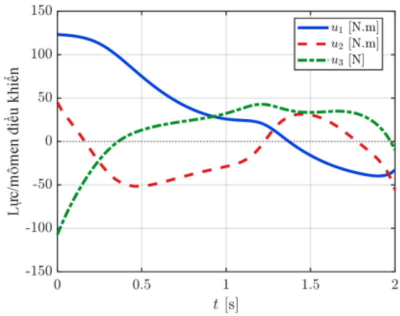
Hình 3.4. Đồ thị biến khớp và vận tốc khớp khâu 1



Hình 3.5. Đồ thị biến khớp và vận tốc khớp khâu 2

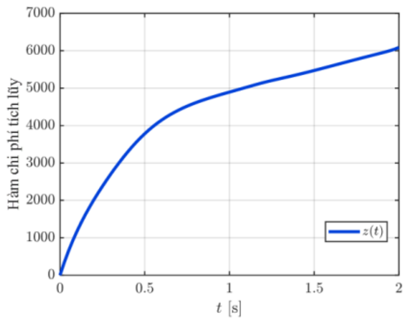


Hình 3.6. Đồ thị biến khớp và vận tốc khớp khâu 3



Hình 3.7. Đồ thị các lực và mômen điều khiển

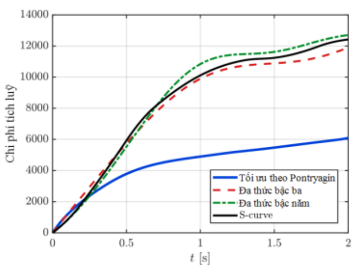
Figures 3.4 to 3.6 show the joint angles and the corresponding joint velocities of each link. The joint velocities vary continuously, without large oscillations, and satisfy the zero-velocity conditions at the initial and final times. The control forces and moments shown in Figure 3.7 also vary continuously, without large oscillations or abrupt changes. The monotonicity of the objective function in Figure 3.8 indicates the accumulation of the control cost.



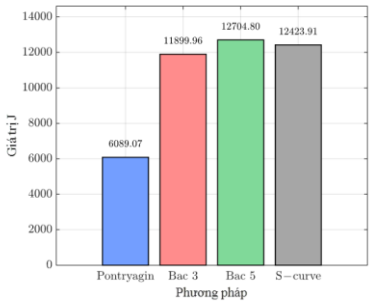
Hình 3.8. Đồ thị hàm chi phí tích luỹ

3.1.5. Quantitative comparison with reference motion laws

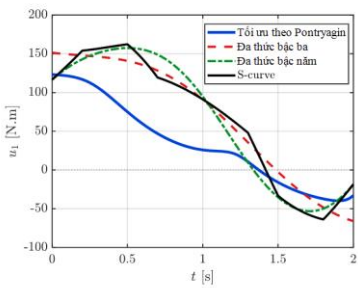
From Figure 3.9, it can be observed that the cumulative cost functions of all four alternatives increase over time, which is consistent with the form of the objective function constructed from the squares of the control forces/moments.



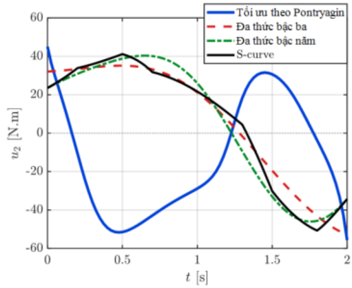
Hình 3.9. Đồ thị hàm chi phí tích lũy của các quy luật chuyển động



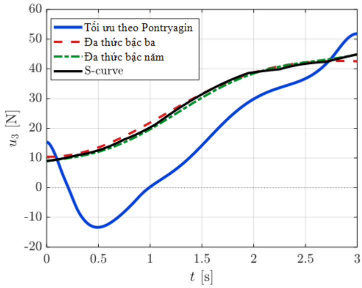
Hình 3.10. Biểu đồ so sánh giá trị hàm mục tiêu



Hình 3.11. Đồ thị mômen u_1 của các quy luật chuyển động



Hình 3.12. Đồ thị mômen u_2 của các quy luật chuyển động



Hình 3.13. Đồ thị lực u_3 của các quy luật chuyển động

Bảng 3.1. So sánh giá trị hàm mục tiêu giữa nghiệm tối ưu và các quy luật chuyển động đối chứng trong chuyển động đến điểm đích

Quy luật chuyển động	**Giá trị hàm mục tiêu J**	**Mức giảm của nghiệm tối ưu so với các quy luật đối chứng**
Tối ưu theo Pontryagin	6089,07	
Đa thức bậc ba	11899,96	48,83%
Đa thức bậc năm	12704,80	52,07%
S-curve	12423,91	50,99%

The results shown in Figure 3.10 and Table 3.1 indicate that the optimal control solution based on Pontryagin's principle yields the smallest value of the objective function. Compared with the reference motion laws, the optimal solution reduces the objective function value by approximately 48.83% to 52.07%.

3.2. Optimal control of manipulators in cyclic point-to-point motion

3.2.1. Problem statement

The complete working cycle of the manipulator consists of two phases:

- In the first phase, the manipulator carries a payload and moves from the initial position to the final position.
- In the second phase, the unloaded manipulator moves from the final position back to the initial position or to a specified working configuration.

The requirement is to determine the control laws u(t) corresponding to the entire cycle so that the manipulator completes both the forward and return strokes under the prescribed boundary conditions, while also satisfying the required optimality criterion.

3.2.2. Formulation of the optimal control problem

In the first phase, the end-effector of the manipulator moves from the initial position $M_0\left(x_0, y_0\right)$ corresponding to the initial configuration $\mathbf{q}\left(0\right)=\mathbf{q}_0$ to the final position $M_d\left(x_d, y_d\right)$, corresponding to the configuration $\mathbf{q}\left(t_{f1}\right)=\mathbf{q}_d$ at the prescribed time $t=t_{f1}$, with the payload $m=m_d$. In the second phase, the manipulator moves without payload from the position M_d back to the position

$M_f \equiv M_0$, corresponding to the configuration $\mathbf{q}\left(t_f\right)=\mathbf{q}_0$ within the time interval t_{f2}. The objective function selected for each phase of the motion is:

$$J=\int_0^{t_f} f_0 \mathrm{d}t \rightarrow \min \; ; \; f_0=\frac{1}{2}\left(u_1^2+u_2^2+u_3^2\right) \tag{1.21}$$

3.2.3. Simulation results for optimal control

a) Simulation setup

Numerical simulations are performed using the parameters selected as in Section 3.1.4, with $t_{f_1}=t_{f_2}=2\left[\mathrm{s}\right], t_f=t_{f_1}+t_{f_2}=4\left[\mathrm{s}\right]$. The boundary conditions of the cyclic motion problem are selected as follows:

The boundary conditions of the forward phase are:

$$q_1(0)=0\left[\mathrm{rad}\right]; q_2(0)=0,2\left[\mathrm{rad}\right]; q_3(0)=0\left[\mathrm{m}\right];$$

$$q_1\left(t_{f_1}\right)=1,4\left[\mathrm{rad}\right]; q_2\left(t_{f_1}\right)=1,0\left[\mathrm{rad}\right]; q_3\left(t_{f_1}\right)=0,5\left[\mathrm{m}\right]$$

$$\dot{q}_1\left(0\right)=\dot{q}_2\left(0\right)=\dot{q}_3\left(0\right)=\dot{q}_1\left(t_{f_1}\right)=\dot{q}_2\left(t_{f_1}\right)=\dot{q}_3\left(t_{f_1}\right)=0$$

$$z(0)=0$$

The boundary conditions of the return phase are:

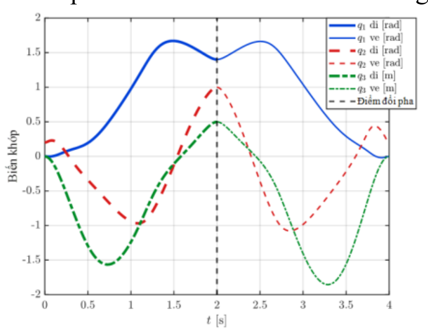
$$q_1\left(t_{f_1}\right)=1,4\ \left[\mathrm{rad}\right]; q_2\left(t_{f_1}\right)=1,0\left[\mathrm{rad}\right]; q_3\left(t_{f_1}\right)=0,5\left[\mathrm{m}\right];$$

$$q_1(t_f)=0\left[\mathrm{rad}\right]; q_2(t_f)=0,2\left[\mathrm{rad}\right]; q_3(t_f)=0\left[\mathrm{m}\right];$$

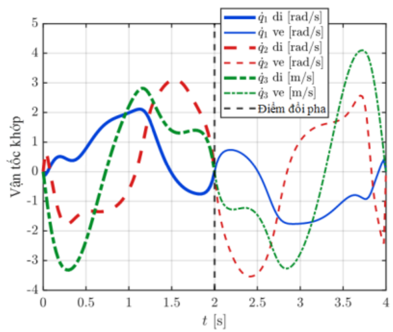
$$\dot{q}_1\left(t_{f_1}\right)=\dot{q}_2\left(t_{f_1}\right)=\dot{q}_3\left(t_{f_1}\right)\dot{q}_1\left(t_f\right)=\dot{q}_2\left(t_f\right)=\dot{q}_3\left(t_f\right)=0;$$

b) Simulation results

From Figure 3.14, it can be observed that the joint variables vary continuously in both the forward and return phases. The joint variables reach the correct configuration at the transition point; thereafter, the manipulator continues the return phase to recover the initial configuration.

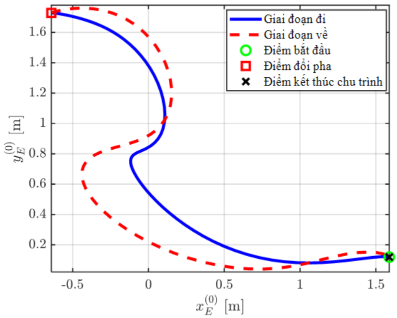


Hình 3.14. Đồ thị các góc khớp giai đoạn đi và về

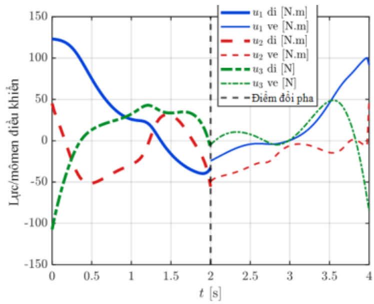


Hình 3.15. Đồ thị các vận tốc khớp giai đoạn đi và về

Figure 3.15 shows that the joint velocities at the start time, at the phase transition point, and at the end of the cycle are all equal to zero.

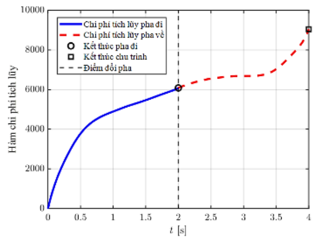


Hình 3.16. Đồ thị vị trí điểm tác động cuối



Hình 3.17. Đồ thị mômen điều khiển giai đoạn đi và về

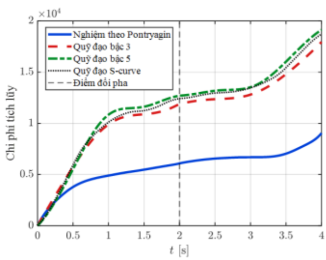
The trajectories of the forward and return phases in Figure 3.16 connect the initial position, the phase transition position, and the final position in accordance with the specified requirements. Figure 3.17 shows that the control moments vary continuously in each phase and do not exhibit abnormal oscillations. Figure 3.18 presents the cumulative value of the objective function over the entire cycle.



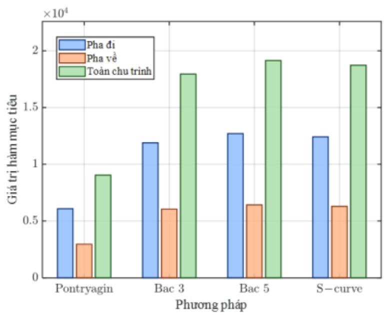
Hình 3.18. Đồ thị tích luỹ hàm mục tiêu giai đoạn đi và về

3.2.4. Quantitative comparison with reference motion laws

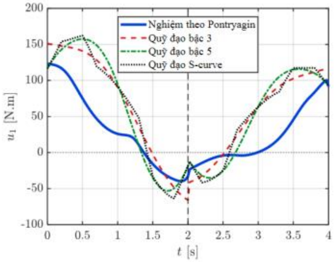
From Figure 3.19, it can be observed that the cumulative cost functions of the alternatives all increase over time, which is consistent with the form of the objective function constructed from the squares of the control moments. According to Table 3.2, the optimal control solution reduces the objective function value for the entire cycle by approximately 49.59% compared with the cubic polynomial motion law, 52.70% compared with the quintic polynomial motion law, and 51.66% compared with the S-curve motion law.



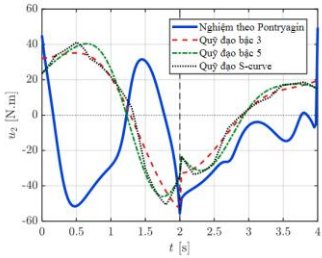
Hình 3.19. Đồ thị hàm chi phí tích lũy của các quy luật chuyển động



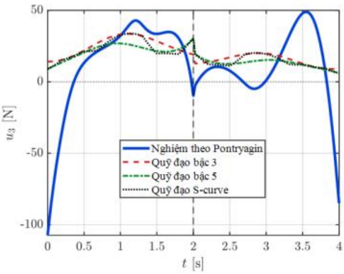
Hình 3.20. Biểu đồ so sánh giá trị hàm mục tiêu



Hình 3.21. Đồ thị mômen u_1 của các quy luật chuyển động



Hình 3.22. Đồ thị mômen u_2 của các quy luật chuyển động



Hình 3.23. Đồ thị mômen u_3 của các quy luật chuyển động

Bảng 3.2. So sánh giá trị hàm mục tiêu giữa nghiệm tối ưu và các quy luật chuyển động đối chứng trong chuyển động điểm-điểm theo chu trình

Quy luật chuyển động	Giá trị hàm mục tiêu J			Mức giảm của nghiệm tối ưu so với các quy luật đối chứng trong toàn bộ chu trình
	Pha đi	Pha về	Chu trình	
Tối ưu theo Pontryagin	6089,07	2960,65	9049,72	
Đa thức bậc ba	11899,96	6050,73	17950,68	49,59%
Đa thức bậc năm	12704,80	6429,21	19134,01	52,70%
S-curve	12423,91	6297,60	18721,51	51,66%

Conclusion of Chapter 3

From the contents of Chapter 3, the following conclusions can be drawn:

1- The methodological framework developed in Chapter 2 can be effectively applied to the optimal control problem of industrial manipulators in point-to-point motion.

2- The application of Pontryagin's Maximum Principle makes it possible to transform the optimal control problems under consideration into systems of differential equations with boundary conditions, in which the state variables, costate variables, and optimal control laws are determined simultaneously.

3- The numerical simulation results show that the obtained optimal solutions ensure that the manipulator performs the required motions correctly. A quantitative comparison of the optimization criterion was conducted against reference motion laws, including cubic polynomial, quintic polynomial, and S-curve laws. For the target-point motion problem, the optimal solution reduces the objective function value by 48.83%, 52.07%, and 50.99%, respectively, compared with the reference laws. For the cyclic point-to-point motion problem, the objective function value over the entire cycle is reduced by 49.59%, 52.70%, and 51.66%, respectively.

CHAPTER 4. OPTIMAL CONTROL OF INDUSTRIAL MANIPULATORS SUBJECT TO END-EFFECTOR TRAJECTORY CONSTRAINTS

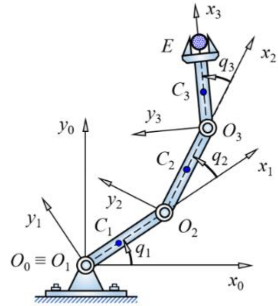
4.1. Optimal control of manipulators subject to end-effector trajectory constraints

4.1.1. Problem statement

Consider a planar manipulator with three revolute links, as shown in Figure 4.1.

The problem is to optimize the control effort of the manipulator by determining the optimal driving forces/moments at the joints, namely the control variables, such that the end-effector E, with coordinates $\left(x_E, y_E\right)$ in the coordinate system $O_0x_0y_0$ moves in the vertical plane along a prescribed trajectory, which is a straight line defined by

$$G \triangleq y_E + ax_E - b = 0 \tag{1.22}$$



Hình 4.1. Tay máy phẳng ba khâu quay (RRR)

where a, b re prescribed coefficients.

4.1.2. Establishment of the dynamic equations of the constrained manipulator

Taking the time derivative of G gives:

$$G_1 \triangleq \frac{\mathrm{d}G}{\mathrm{d}t} = \sum_{i=1}^{3} \frac{\partial G}{\partial q_i}\dot{q}_i = \sum_{i=1}^{3} d_i\dot{q}_i = 0;\ d_i = \frac{\partial G}{\partial q_i} \tag{1.23}$$

From this, the matrix **D** is obtained as follows:

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & -\dfrac{d_1}{d_3} \\ 0 & 1 & -\dfrac{d_2}{d_3} \end{bmatrix} \tag{1.24}$$

4.1.3. Formulation of the optimal control problem

The requirement is to determine the control variables $\mathbf{u}(t)$ and the corresponding joint-angle trajectory $\mathbf{q}(t)$ such that the end-effector of the manipulator moves from the initial position $M_0\left(x_0, y_0\right)$, corresponding to the joint angles $\mathbf{q}_0 = \mathbf{q}(0)$, , along the straight-line trajectory defined by Eq. (4.1), to the target position $M_f\left(x_f, y_f\right)$, corresponding to the joint angles $\mathbf{q}_f = \mathbf{q}\left(t_f\right)$. The quantities $\mathbf{q}_0, \mathbf{q}_f, t_f$ are prescribed, and the following optimality criterion is satisfied:

$$J = \int_0^{t_f} f_0 \mathrm{d}t \rightarrow \min\ ;\ f_0 = \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2\right) \tag{1.25}$$

4.1.4. Numerical simulation results

a) Simulation setup

Numerical simulations are performed using the following selected parameters::

$$m_1 = 4,2;\ m_2 = 3,0;\ m_3 = 2,88;\ m_d = 1,5\ [\mathrm{kg}]$$

$$L_1 = 0,8;\ L_2 = 0,6;\ L_3 = 0,5\ \ [\mathrm{m}];\ c_1 = 0,4;\ c_2 = 0,3;\ c_3 = 0,25\ [\mathrm{m}]$$

$$J_1 = 0,224;\ J_2 = 0,09;\ J_3 = 0,06\ \left[\mathrm{kg.m^2}\right]$$

$$b_1 = 3; b_2 = 5; b_3 = 5\left[\mathrm{N.m.s/rad}\right]; g = 10\left[\mathrm{m/s^2}\right];\ t_f = 2[s]$$

The boundary conditions:

$$q_1(0) = 0[\mathrm{rad}]; q_2(0) = \frac{\pi}{6}\ [\mathrm{rad}]; q_3(0) = \frac{\pi}{6}[\mathrm{rad}]$$

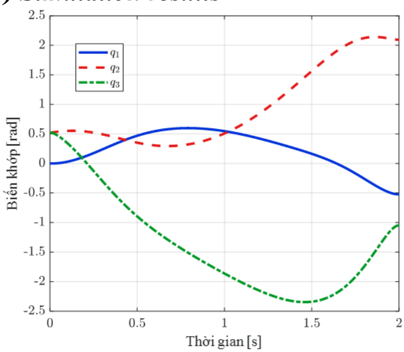
$$q_1(t_f) = -\frac{\pi}{6}[\mathrm{rad}]; q_2(t_f) = \frac{2\pi}{3}\ [\mathrm{rad}];\ q_3(t_f) = -\frac{\pi}{3}[\mathrm{rad}];$$

$$\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = 0$$

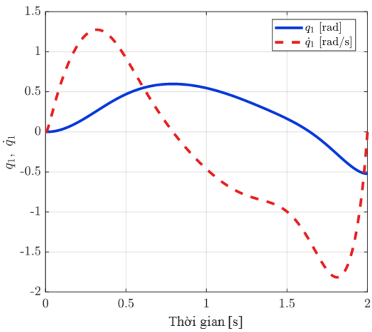
$$\dot{q}_1(t_f) = \dot{q}_2(t_f) = \dot{q}_3(t_f) = 0$$

The objective functional is $z = \int_0^t f_0 \mathrm{d}t\ ;\ z(0) = 0$.

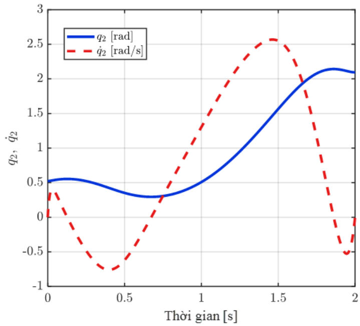
b) Simulation results



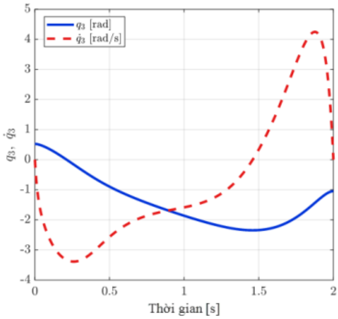
Hình 4.2. Đồ thị các biến khớp



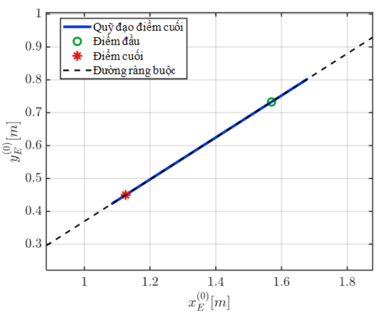
Hình 4.3. Đồ thị biến khớp và vận tốc khớp khâu 1



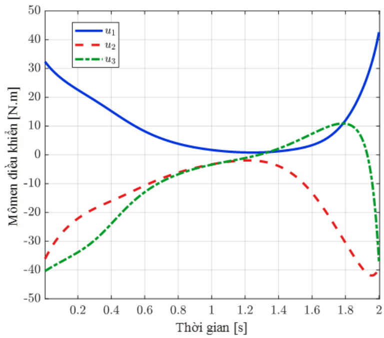
Hình 4.4. Đồ thị biến khớp và vận tốc khớp khâu 2



Hình 4.5. Đồ thị biến khớp và vận tốc khớp khâu 3



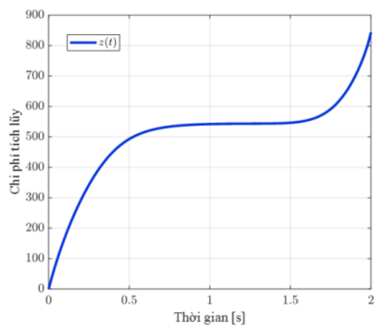
Hình 4.6. Đồ thị quỹ đạo chuyển động điểm cuối



Hình 4.7. Đồ thị các lực/mômen điều khiển

In Figure 4.2, the joint variables vary continuously over time, without jumps or abnormal oscillations. Figures 4.3, 4.4, and 4.5 show the detailed variation laws of the joint variables and the corresponding joint velocities. It can be observed that the velocities of all joints are equal to zero at the initial and final times, which is consistent with the prescribed boundary conditions.

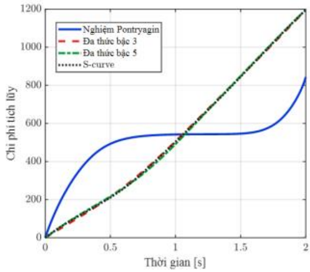
Figure 4.6 shows that the end-effector of the manipulator moves from the initial point to the final point while following the prescribed geometric constraint. Figure 4.7 shows that the control signals u1, u2, and u3 vary continuously throughout the simulation process. Figure 4.8 presents the cumulative value of the objective function z(t), which increases rapidly in the initial phase, then increases more slowly in the middle phase, and continues to increase more strongly in the final phase. This behavior is consistent with the variation of the control forces/moments shown in Figure 4.7.



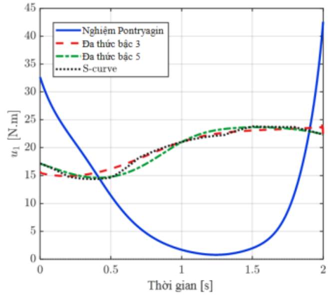
Hình 4.8. Đồ thị hàm mục tiêu

4.1.5. Quantitative comparison with reference motion laws

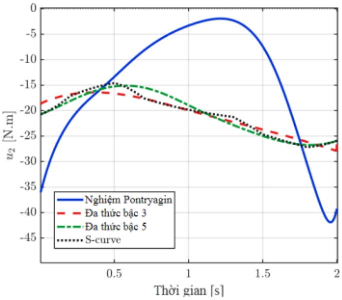
Figure 4.9 shows that the cumulative cost curve of the Pontryagin-based solution is clearly lower than those of the reference motion laws over most of the motion process.



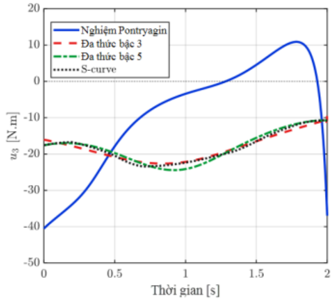
Hình 4.9. Đồ thị hàm chi phí tích lũy của các quy luật chuyển động



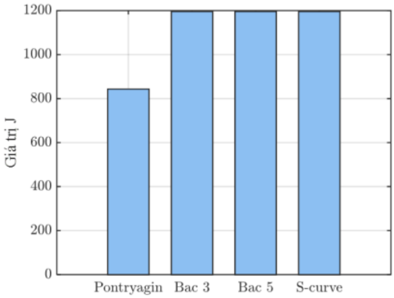
Hình 4.10. Đồ thị mômen u_1 của các quy luật chuyển động



Hình 4.11. Đồ thị mômen u_2 của các quy luật chuyển động



Hình 4.12. Đồ thị mômen u_3 của các quy luật chuyển động



Hình 4.13. Biểu đồ so sánh giá trị hàm mục tiêu

Bảng 4.1. So sánh giá trị hàm mục tiêu giữa nghiệm tối ưu và các quy luật chuyển động đối chứng trong chuyển động chịu ràng buộc quỹ đạo điểm cuối

Quy luật chuyển động	**Giá trị hàm mục tiêu J**	**Mức giảm của nghiệm tối ưu so với các quy luật đối chứng**
Tối ưu theo Pontryagin	843,56	
Đa thức bậc ba	1195,88	29,46%
Đa thức bậc năm	1195,89	29,46%
S-curve	1196,02	29,47%

The control signals of all four methods shown in Figures 4.10 to 4.12 vary smoothly, without abnormal peaks. In general, the Pontryagin-based solution does not minimize the amplitude at each individual actuator separately, but coordinates all three control variables to reduce the cumulative cost function.

The results shown in Figure 4.13 indicate that the Pontryagin-based optimal solution significantly reduces the control cost compared with the reference motion laws. Specifically, the optimal solution reduces the energy by approximately 29.46% compared with the cubic polynomial motion law, approximately 29.46% compared with the quintic polynomial motion law, and approximately 29.47% compared with the S-curve motion law. This demonstrates that the optimal control solution not only satisfies the boundary conditions and the end-effector trajectory constraint, but also achieves better performance according to the selected optimality criterion.

4.2. Optimal control of manipulators along a prescribed trajectory in cyclic motion

4.2.1. Problem statement

Consider the planar RRP manipulator of type 2, as described in Section 4.1. The motion of the manipulator is performed over a cycle consisting of a forward phase and a return phase. In the forward phase, the manipulator carries a payload m at the end-effector and moves from the initial point to the target point along a prescribed trajectory. In the return phase, the manipulator moves from the target point back to the initial position without carrying a payload, while still following the prescribed trajectory of the end-effector..

4.2.2. Formulation of the optimal control problem

In the forward phase, the end-effector of the manipulator moves from the initial position $M_0(x_0, y_0)$, corresponding to the initial configuration $\mathbf{q}(0) = \mathbf{q}_0$, over the time interval $0 \le t \le t_{f1}$ to the final position $M_d(x_d, y_d)$, corresponding to the configuration $\mathbf{q}(t_{f1}) = \mathbf{q}_d$, with the payload $m = m_d$. In the return phase, the manipulator moves without payload, and the end-effector moves from M_d back to $M_f \equiv M_0$, corresponding to the configuration $\mathbf{q}(t_f) = \mathbf{q}_0$ over the time interval $t_{f1} \le t \le t_{f2}$. The total motion time of one cycle is $t_f = t_{f1} + t_{f2}$.

The objective function selected for each phase of the motion is

$$J = \int_0^{t_f} f_0 \mathrm{d}t \to \min \; ; \quad f_0 = \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2\right) \qquad (1.26)$$

4.2.3. Numerical simulation results

a) Simulation setup

Các điều kiện biên được chọn như sau:

Boundary conditions of the forward phase ($0 \le t \le t_{f1} = 2[\mathrm{s}]$)

$$q_1(0) = 0[\mathrm{rad}]; q_2(0) = \frac{\pi}{6}[\mathrm{rad}]; q_3(0) = \frac{\pi}{6}[\mathrm{rad}]$$

$$q_1(t_{f_1}) = -\frac{\pi}{6}[\mathrm{rad}]; q_2(t_{f_1}) = \frac{2\pi}{3}[\mathrm{rad}]; \; q_3(t_{f_1}) = -\frac{\pi}{3}[\mathrm{rad}];$$

$$\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = 0$$

$$\dot{q}_1(t_{f_1}) = \dot{q}_2(t_{f_1}) = \dot{q}_3(t_{f_1}) = 0$$

- Boundary conditions of the return phase ($t_{f1} \le t \le t_{f2}$)

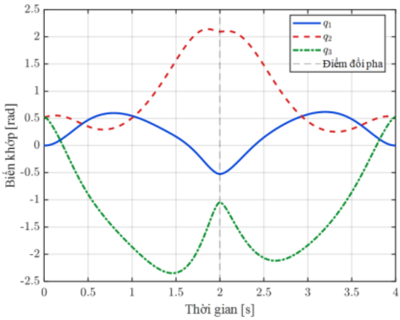
$$q_1(t_{f1}) = -\frac{\pi}{6}[\text{rad}];\, q_2(t_{f1}) = \frac{2\pi}{3}[\text{rad}];\, q_3(t_{f1}) = -\frac{\pi}{3}[\text{rad}];$$

$$q_1(t_f) = 0[\text{rad}];\, q_2(t_f) = \frac{\pi}{6}[\text{rad}];\, q_3(t_f) = \frac{\pi}{6}[\text{rad}];$$

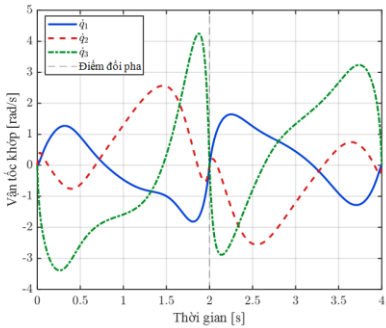
$$\dot{q}_1\left(t_{f_1}\right) = \dot{q}_2\left(t_{f_1}\right) = \dot{q}_3\left(t_{f_1}\right) = 0;\, \dot{q}_1\left(t_f\right) = \dot{q}_2\left(t_f\right) = \dot{q}_3\left(t_f\right) = 0$$

b) Simulation results

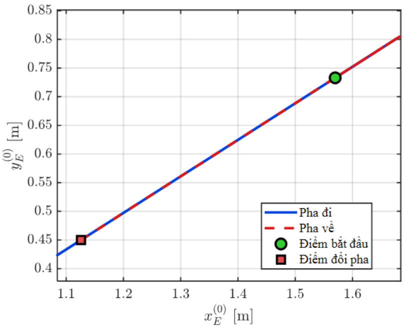
Figure 4.14 shows that the joint variables vary continuously over time during both the forward and return strokes. The values of the generalized coordinates at the beginning and end of each stroke are consistent with the prescribed boundary conditions.



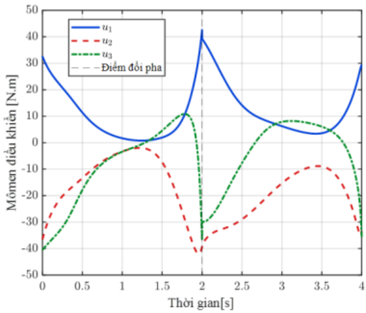
Hình 4.14. Đồ thị các biến khớp giai đoạn đi và về



Hình 4.15. Đồ thị các vận tốc khớp giai đoạn đi và về



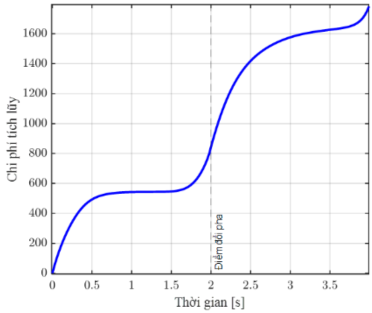
Hình 4.16. Đồ thị quỹ đạo điểm cuối



Hình 4.17. Đồ thị mômen điều khiển giai đoạn đi và về

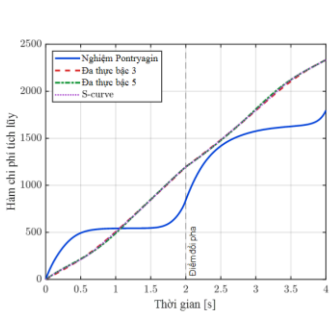
Figure 4.15 shows the joint velocities over the entire cycle. The joint velocities satisfy the zero-velocity condition at the initial time, at the phase transition point, and at the end of the cycle..

The trajectory results of the end-effector in the working plane shown in Figure 4.16 indicate that the end-effector trajectories in both the forward and return phases lie on the prescribed geometric constraint. This shows that the formulation of the optimal control problem for each phase of the cycle still ensures satisfaction of the trajectory constraint at the end-effector. Figure 4.17 presents the corresponding optimal control forces during the two strokes. It can be observed that the control forces vary continuously over time, without large abrupt changes. The value of the objective function shown in Figure 4.18 increases monotonically over time, which is consistent with the nature of the objective function.

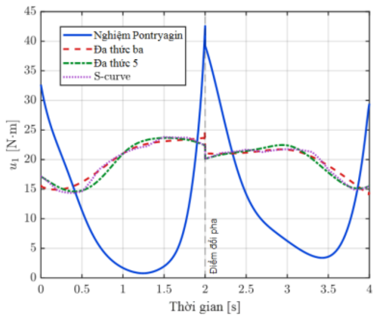


Hình 4.18. Đồ thị hàm chi phí giai đoạn đi và về

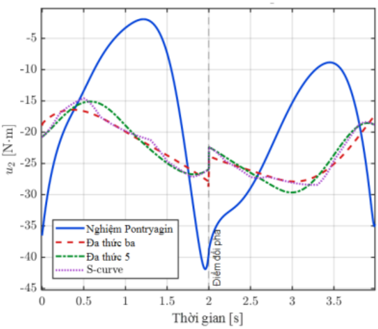
4.2.4. Quantitative comparison with reference motion laws



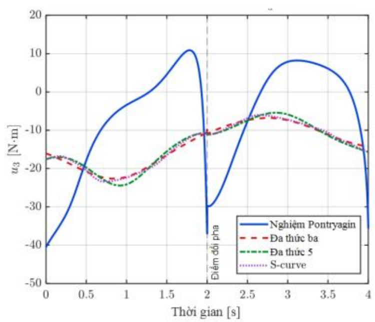
Hình 4.19. Đồ thị hàm chi phí tích lũy của các quy luật chuyển động



Hình 4.20. Đồ thị mômen u_1 của các quy luật chuyển động



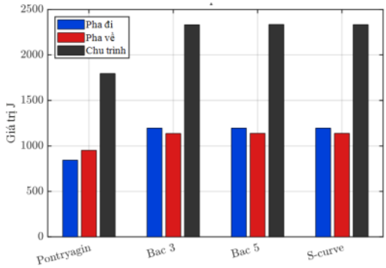
Hình 4.21. Đồ thị mômen u_2 của các quy luật chuyển động



Hình 4.22. Đồ thị mômen u_3 của các quy luật chuyển động

Figure 4.19 presents the variation of the cumulative cost function over time for the Pontryagin-based optimal solution and the reference motion laws. It can be observed that the cumulative cost curve of the Pontryagin-based solution is clearly lower than those of the reference laws over most of the cycle time, including both the forward and return phases. The three control-variable plots shown in Figures

4.20 to 4.22 indicate that the control signals of all methods are continuous and do not exhibit abnormal peaks caused by trajectory singularities, as in the previous case.



Hình 4.23. Biểu đồ so sánh giá trị hàm mục tiêu toàn chu trình

Bảng 4.2. So sánh giá trị hàm mục tiêu giữa nghiệm tối ưu và các quy luật chuyển động đối chứng trong chuyển động chịu ràng buộc quỹ đạo điểm cuối theo chu trình

Quy luật chuyển động	**Giá trị hàm mục tiêu J toàn chu trình**	**Mức giảm của nghiệm tối ưu so với các quy luật đối chứng trong toàn bộ chu trình**
Tối ưu theo Pontryagin	1795,79	
Đa thức bậc ba	2332,38	23,01%
Đa thức bậc năm	2335,12	23,10%
S-curve	2334,06	23,06%

The results in terms of the J value show that the objective function value over the entire cycle for the Pontryagin-based optimal solution is reduced by 23.01% compared with the cubic polynomial motion law, 23.10% compared with the quintic polynomial motion law, and 23.06% compared with the S-curve motion law. This demonstrates that the optimal control solution achieves clear quantitative effectiveness according to the selected criterion of control-effort optimization.

Conclusion of Chapter 4

From the contents and results obtained in Chapter 4, the following conclusions can be drawn:

1- The methodological framework of the dissertation can be further extended from point-to-point motion problems to optimal control problems of industrial manipulators subject to end-effector trajectory constraints.

2- The application of Pontryagin's Maximum Principle to manipulator models subject to trajectory constraints shows that the dissertation's approach is not only suitable for unconstrained mechanical systems but can also handle mechanical systems subject to geometric constraints throughout the motion process.

3- The quantitative comparisons with the reference motion laws further clarify the effectiveness of the Pontryagin-based optimal solution. For the problem of single motion subject to an end-effector trajectory constraint, the objective function value of the optimal solution is reduced by approximately 29.46%, 29.46%, and 29.47%, respectively, compared with the reference motion laws. For the cyclic motion problem, the objective function value over the entire cycle of the optimal solution is reduced by approximately 23.01%, 23.10%, and 23.06%, respectively.

GENERAL CONCLUSIONS AND FURTHER RESEARCH DIRECTIONS

General conclusion

From the contents and results obtained in the dissertation, the following conclusions can be drawn:

1- The dissertation has developed a relatively unified methodological framework for the class of problems involving dynamic analysis and motion optimization of industrial manipulators.

2- For the point-to-point motion problem, the dissertation has formulated, developed, and solved the optimal control problem for two representative cases: motion to a target point and cyclic motion. The obtained results show that the established dynamic models are appropriate, the optimization problem is properly formulated, and the resulting optimal solution ensures that the manipulator performs the required motion correctly, with joint variables, joint velocities, and control signals varying continuously and in a mechanically reasonable manner.

3- For the motion problem subject to end-effector trajectory constraints, the dissertation has extended the methodological framework from unconstrained systems to systems subject to geometric constraints. The research results show that the equations of motion of the constrained system can be established in a form suitable for applying Pontryagin's principle, thereby making it possible to determine the optimal control law that ensures the end-effector follows the prescribed trajectory throughout the entire motion process.

4- The quantitative comparisons with reference motion laws, such as cubic polynomial, quintic polynomial, and S-curve laws, show that the Pontryagin-based optimal solution significantly reduces the value of the objective function.

5- In terms of contributions, the dissertation does not merely apply existing theoretical results to specific models, but organizes and develops a systematic approach for the class of problems involving dynamics and optimal control of industrial manipulators. The planar three-degree-of-freedom manipulator models investigated in the dissertation serve to illustrate and verify the proposed methodological framework.

Further research directions

1- Extend the research direction of the dissertation to robot manipulator models with spatial structures and a larger number of degrees of freedom.

2- Further investigate and incorporate into the dynamic model influencing factors such as environments with obstacles, uncertainties in the dynamic model, and the elasticity of links and joints.

3- Conduct experimental verification on a specific manipulator model.